

On the angular momentum dependence of nuclear level densities

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ABSTRACT

Angular momentum dependence of nuclear level densities at finite temperatures are investigated in the static path approximation (SPA) to the partition function using a cranked quadrupole interaction Hamiltonian in the following three schemes: (i) cranking about x-axis, (ii) cranking about z-axis and (iii) cranking about z-axis but correcting for the orientation fluctuation of the axis. Performing numerical computations for an *sd* and a *pf* shell nucleus, we find that the x-axis cranking results are satisfactory for reasonably heavy nuclei and this offers a computationally faster method to include the angular momentum dependence at high temperatures in the SPA approach. It also appears that at high spins inclusion of orientation fluctuation correction would be important.

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1. Introduction

The level density is one of the important physical quantities appearing in the statistical analysis of nuclear reactions[1]. It also provides a reasonable basis for testing the applicability of the various approximations used for many particle systems. The most commonly used mean field approximations (MFA), like Hartree-Fock[2], Hartree-Fock Bogoliubov[3, 4] have been successful in describing the structure of the nuclei upto a very high spin state along the yrast line as well as for the low-lying excited states. However, as one moves far away from the yrast line, the theoretical predictions based on the MFA show an inconsistency with the experimental data on the shape transitions[5], strength function for giant dipole resonance(GDR)[6], angular distribution of GDR γ -rays[7] etc. A simplest way to improve upon this inconsistency is to account for the statistical fluctuations associated with various degrees of freedom (e.g. pairing and quadrupole degrees of freedom) quadrupole and pairing degrees of freedom) caused by the symmetry breaking[8] required to incorporate the correlations. As a first step one includes the effects of fluctuations by considering an isothermal probability distribution as a function of an appropriate set of phase variables. In this way one finds a remarkable improvement in understanding the experimental data. However, Alhassid and Bush have shown recently[9] that this method of including statistical fluctuations in the MFA leads to a serious problem of overestimating the values of the level density. On the other hand, they find that the static path approximation(SPA) [10, 11, 12] which provides a natural framework to deal with the statistical fluctuations yields an almost exact value of the level density at high temperatures.

The SPA has been applied for the model studies of the nuclei at finite temperatures. All these model studies indicate that SPA becomes almost exact at high temperatures. Furthermore, the results at low temperatures can

be improved by including the contribution of Fock term[13] or the quantum correlation corrections in the random phase approximation(RPA) method [14, 15]. For a realistic application, however, one must restore the broken symmetries in order to characterize the system by a set of physically observable quantum numbers, e.g., angular momentum, particle numbers etc. In ref. [16], the usual exact projection method has been employed to conserve the particle number at finite temperature. Recently, this method has been extended to project out a general symmetry from the partition function involving a one-body statistical operator[17]. However, this method is rather involved to be numerically tractable for heavy nuclei. For instance, the projection of an exact angular momentum (i.e. $SO(3)$ symmetry group) state would require an additional three dimensional integration besides those over the phase variables appearing in the SPA representation of partition function(e.g. see eq. (9)). On the other hand, by applying SPA to a standard J_x -cranking Hamiltonian, in refs. [18, 19] we have studied the thermal properties of a medium heavy rotating nucleus, ^{64}Zn . It is shown that the results for the energy, level density, moment of inertia etc. provide a reasonable physical insight at high spins and temperatures. In the present work we further investigate the angular momentum dependence of the level densities at finite temperatures in SPA using cranked quadrupole Hamiltonian in the following three schemes: (i) cranking about x-axis as just mentioned, (ii) cranking about z-axis and (iii) cranking about z-axis but correcting for the orientation fluctuation of the axis.

In the next section we present the basic expressions for computing the level densities in different cranking schemes mentioned above. Section 3 contains the numerical details and discussions on the results obtained for an *sd* (^{24}Mg) and a *pf* (^{64}Zn) shell nucleus. Finally, a brief summary and

conclusions will be presented in section 4.

2. Theoretical framework

The grand canonical partition function for a cranked Hamiltonian is given as:

$$Z(\mu_p, \mu_n, \omega, T) = \text{Tr} e^{-(\hat{H} - \mu_p \hat{N}_p - \mu_n \hat{N}_n - \omega \hat{J}_i)/T} \quad (1)$$

where i denotes the cranking axis in laboratory frame, the trace is taken over all the many-body states of the system described by the Hamiltonian \hat{H} . The quantities μ_p , μ_n and ω are essentially the Lagrange multipliers used to constrain the average number of protons N_p , neutrons N_n and the component of total angular momentum along the cranking axis, i.e, J_i . The Hamiltonian \hat{H} considered here is

$$\hat{H} = \hat{H}_o - \frac{1}{2} \chi \sum_{\mu} \hat{Q}_{\mu} \hat{Q}_{\mu}^{\dagger} \quad (2)$$

where, $\hat{H}_o = \sum_i h_o(i)$ stands for the spherical part of the Hamiltonian with $h_o(i)$ corresponding to the basis state single-particle (sp) energies, χ is the quadrupole interaction strength and $\hat{Q}_{\mu} = r^2 Y_{2\mu}$ is the quadrupole moment operator.

In SPA, the eq.(1) can be written as follows,

$$Z(\mu_p, \mu_n, \omega, T) = \text{Tr} \hat{\mathcal{D}}(\mu_p, \mu_n, \omega, T) \quad (3)$$

where, $\hat{\mathcal{D}}$ is a one-body static path statistical operator as given below,

$$\hat{\mathcal{D}} = \int \mathcal{D}(\sigma) e^{-(\hat{H}'(\sigma) - \mu_p \hat{N}_p - \mu_n \hat{N}_n - \omega \hat{J}_i)/T} \quad (4)$$

where, $\int \mathcal{D}(\sigma)$ denotes the integration over the temperature independent (or static) paths in the space of auxilliary fields, $\sigma \equiv \{\sigma_1, \sigma_2, \dots\}$ and $\hat{H}'(\sigma)$ is a

one-body Hamiltonian. Using ref. [10] we have

$$\int \mathcal{D}(\sigma) = \left(\frac{\chi}{2\pi T}\right)^{5/2} \int \prod_{\mu} d\sigma_{\mu} e^{-\frac{\chi}{2T} \sum_{\mu} |\sigma_{\mu}|^2} \quad (5)$$

$$\hat{H}'(\sigma) = \hat{H}_o - \chi \sum_{\mu} (-1)^{\mu} \sigma_{\mu} \hat{Q}_{-\mu} \quad (6)$$

The above eqs. (5) and (6) can be represented conveniently in the intrinsic coordinate system using the eqs. (7 and (8) given below,

$$\sigma_{\mu} = \sqrt{\frac{M\omega_o}{\chi}} \sum_{\nu} a_{\nu} D_{\nu\mu}^2(\phi, \theta, \psi) \quad (7)$$

where, $a_{\pm 1} = 0$, $a_{\pm 2} = \beta \sin \gamma / \sqrt{2}$ and $a_o = \beta \cos \gamma$ and D is the standard Wigner's function.

$$J_i = \sum_j R_{i,j} J_j(\phi, \theta, \psi) \quad (8)$$

where, R is the transformation matrix which essentially comprises of usual rotation D matrices and the indices i and j denote the component in the lab and the intrinsic frame, respectively. Let us first consider the third scheme (as discussed above). Using eqs. (3) - (8), we have (see ref. [10])

$$\begin{aligned} Z &= 2\pi \left(\frac{\alpha}{2\pi T}\right)^{5/2} \int_o^{2\pi} d\psi \int_o^{\pi} d\theta \sin \theta \int_o^{\beta_{max}} d\beta \beta^4 \\ &\times \int_o^{\pi/3} d\gamma | \sin 3\gamma | e^{-\frac{\alpha\beta^2}{2T}} \text{Tr} e^{-(H'^{\omega} - \mu_p \hat{N}_p - \mu_n \hat{N}_n)/T} \end{aligned} \quad (9)$$

In the above $\alpha = \frac{M^2 \omega_o^2}{\chi}$. M is the nucleon mass, $\chi = 70 A^{-1.4}$ MeV taken from ref. [21], $\omega_o = 41/A^{1/3}$ MeV and $H'^{\omega} = \sum_i h^{\omega}(i)$

$$\begin{aligned} \hat{h}'^{\omega} &= \hat{h}_o - M\omega_o^2 r^2 \beta [\cos \gamma Y_{2,0} + \frac{1}{\sqrt{2}} \sin \gamma (Y_{2,2} + Y_{2,-2})] \\ &\quad - \omega \{ \cos \theta j_z - \sin \theta \cos \psi j_x + \sin \theta \sin \psi j_y \} \end{aligned} \quad (10)$$

The partition function for other two cases can be obtained simply by replacing the coefficient of ω (curly bracket) in eq. (10) by j_x or j_z . We now give the basic expressions used in computing the energy and level density in terms of the partition function for case (ii) and (iii).

$$E = T^2 \frac{\partial \ln Z}{\partial T} + \mu_p N_p + \mu_n N_n + \omega M \quad (11)$$

where,

$$N_{p,n} = T \frac{\partial \ln Z}{\partial \mu_{p,n}} \quad (12)$$

and

$$M = T \frac{\partial \ln Z}{\partial \omega} \quad (13)$$

is the component of the total angular momentum along the z-axis.

The level density $\rho(E, J)$ at a fixed energy and angular momentum is given by [22, 23]

$$\rho(E, J) = \rho(E, M = J) - \rho(E, M = J + 1) \approx - \frac{\partial \rho(E, M)}{\partial M} \Big|_{M=J+\frac{1}{2}} \quad (14)$$

where, $\rho(E, M)$ is an inverse Laplace transform of the partition function which can be written in a saddle point approximation as

$$\rho(E, M) = \frac{\ln Z e^{(E - \mu_p N_p - \mu_n N_n - \omega M)/T}}{(2\pi)^2 D^{1/2}} \quad (15)$$

D is the determinant of a 4×4 matrix with elements

$$d_{ij} = \frac{\partial^2 \ln Z}{\partial x_i \partial x_j}; \quad x_i = (1/T, -\mu_p/T, -\mu_n/T, -\omega/T) \quad (16)$$

On similar lines, the level density for the case (i) can be obtained directly using eq. (15) with the partition function and its derivatives evaluated at a fixed value of J (see ref. [19] for detail).

Finally, we give expressions for the level density parameter a . Most commonly one uses,

$$E^* = aT^2 \quad (17)$$

where, $E^* = E(T) - E(T = 0)$ is the excitation energy at a given T. However, we know that SPA is not applicable in $T \rightarrow 0$ limit. We therefore use,

$$S = 2aT \tag{18}$$

where, $S = (E - F)/T$ is the entropy with F being the free energy.

3. Numerical details and results

3.1 NUMERICAL DETAILS

In this subsection we shall present a detailed description of the numerical procedure for the calculation of level densities in different cranking schemes as outlined in the previous section. Some details as presented in ref. [19] are not reported here.

We see from the eqs. (11), (12), (14) and (15) that the evaluation of the physical quantities, like energy, particle numbers and level density require the computation of partition function (9) and its first and second derivatives with respect to T , $\mu_{p,n}$ and ω . To begin with, in a suitable model space (e.g. see table 1), we diagonalize the one-body Hamiltonian (10) at mesh points in the space of quadrupole degrees of freedom, i.e. β , γ , θ and ψ (note that eq. (9) is independent of the variable ϕ). Now, the trace appearing in eq. (9) can be obtained for a given value of $\mu_{p,n}$ and ω using the following equation (see also [11, 18]),

$$Tr e^{-(\hat{H}'\omega - \mu_p \hat{N}_p - \mu_n \hat{N}_n - \omega \hat{J}_i)/T} = \prod_i [1 + e^{-(\epsilon_i(\beta, \gamma, \theta, \psi) - \mu_i)/T}] \quad (19)$$

where, ϵ'_i 's are the eigenvalues of the Hamiltonian (10) and the index i runs over all the deformed single particle(sp) orbits for proton as well as neutron. Equation (9) is then computed by performing a numerical integration using eight Gaussian points for each of the variables and taking $\beta_{max} = 0.5$. Having determined Z as a function of $\mu_{p,n}$ and ω at a fixed T , we calculate its first derivatives with respect to $\mu_{p,n}$ and ω and adjust these such that the eqs. (12) and (13) are satisfied for the desired values of $N_{p,n}$ and M , respectively. The energy can be easily calculated using eqs. (11).

Now, we come to the numerical evaluation of the level densities for three different cases discussed in section 2. For the case (i), i.e J_x -cranking, we

first compute the partition function for a desired value of J by adjusting ω such that the r.h.s. of eq. (13) leads to $\langle J_x \rangle = \sqrt{J(J+1)}$ for a given J . The level density $\rho(E, J)$ is then evaluated using this partition function in eq. (15), where $E \equiv E(T)$ represents the average energy at a temperature T . Note here that for the case (i), the integration over θ and ψ appearing in the eq. (9) can be replaced by a factor 4π . For the case (ii), i.e J_z -cranking, it is apparent from the eq. (14) that the quantity $\rho(E, J)$ can be calculated simply by taking the difference between $\rho(E, M = J)$ and $\rho(E, M = J + 1)$ at a fixed value of E . This means that the temperature has to be adjusted such that the energy of the system remains the same at $M = J$ and $J+1$. As an illustration, we show in figure 1, the variation of energy with temperature for $M = 2, 3, 4$ and 5 for ^{24}Mg . The points A and B on the horizontal dashed line indicate the change in the temperature (≈ 0.3 MeV) with M changing from 2 to 3 at $E = -47.5$ MeV. It may be noticed from the figure that the temperature difference for a given E in the high temperature region decreases. However, instead of adjusting T , we adopt a slightly different but relatively faster method which is as follows. To illustrate the method, let us take the calculation of $\rho(E, J = 2)$. We first calculate the values of $\rho(E(T), M)$ for $M = 2$ and 3 at the temperatures $0.4, 0.6, 0.8, \dots, 4.0$ MeV. Then for each value of $\rho(E(T), M = 3)$ we calculate $\rho(E(T'), M = 2)$ using standard interpolation method[24] such that $E_{M=3}(T) = E_{M=2}(T')$. A similar numerical procedure is followed for the evaluation of level density in case (ii). We must add here that the case (iii) requires the diagonalization of a complex one-body Hamiltonian, since $J_y^* = -J_y$, unlike the other two cases. However, the problem of diagonalizing a $n \times n$ complex Hermitian matrix can easily be mapped into a diagonalization of a $2n \times 2n$ real symmetric matrix. [24].

3.2 RESULTS AND DISCUSSIONS

Following the procedure as described above, we have performed the numerical calculations for an *sd* (^{24}Mg) as well as a *pf* shell (^{64}Zn) nucleus. In this subsection we shall present the results for level densities and investigate its angular momentum dependence at finite temperatures using SPA to three different cranking schemes as discussed in the earlier sections. We must mention that the model spaces (see table 1) considered here are precisely the same as used in refs. [25] and [11] for the study of ^{24}Mg and ^{64}Zn at finite temperatures, respectively. From table 1 it is clear that for ^{24}Mg there are 8 valence particles with ^{16}O as a core and for ^{64}Zn there are 24 valence particles with ^{40}Ca as a core.

(i) ^{24}Mg

The level density

Usually the level density is estimated through the level density parameter a with numerical value between $A/8$ - $A/10$ given by the Fermi-gas model. The experimental data[26, 27, 28] suggest that for $A \sim 160$ the parameter a decreases from $A/8$ at low temperatures to $A/13$ at $T \sim 5$ MeV. Using J_x -cranking we have studied here the temperature as well as angular momentum dependence of the inverse level density parameter $K(= A/a)$. Figure 2 shows the variation of K with temperature for a few values of angular momenta, $J = 0, 2, 4$ and 6 . We find that the level density parameter a decreases with temperature and angular momentum. On the average, the value of a decreases from $A/6$ at $T = 1$ MeV to $A/12$ at $T = 3.0$ MeV. More on this will be discussed below.

In figure 3 we have displayed the J_x cranking results for the level density as a function of energy for a few values of angular momenta as indicated.

Similar plots are shown in figures (4) and (5) where the angular momentum dependence is brought through cranking along the z-axis without and with orientation fluctuation corrections(OFC), respectively. We find from these figures that the qualitative behaviour for the level densities obtained in different cranking schemes considered here is more or less the same. To fascilate further discussion we denote the level densites obtained using (i) J_x -cranking, (ii) J_z -cranking and (iii) J_z cranking with OFC by ρ_x , ρ_z and $\tilde{\rho}_z$, respectively. We now analyze the level densities for $J = 0$. It is well known that in J_x -cranking, $J = 0$ simply means $\omega = 0$ (or no cranking). So, strictly speaking, the quantity $\rho_x(E, J = 0)$ essentially corresponds to the intrinsic or unprojected level density. On the other hand, using eq. (14), for the case (ii) and (iii) we have, $\rho(E, J = 0) = \rho(E, M = 0) - \rho(E, M = 1)$. This implies that for $J = 0$, $\rho_x > \rho_{3,z}$ (since $\tilde{\rho}_z(E, M = 0) = \rho_z(E, M = 0) = \rho_x(E, J = 0)$). In addition to this we also note that $\rho_z(E, J = 0) \approx \tilde{\rho}_z(E, J = 0)$ and the difference, $\rho_{xz} = \rho_x(E, J = 0) - \rho_z(E, J = 0)$ is quite large. Moving towards higher values of J , we find that $\tilde{\rho}_z < \rho_z$ for $J = 4$ and 6 and $\rho_x > \rho_z$ for all J . However, the difference $\rho_x - \rho_z$ reduces with increase in J . For example, we find that $\rho_{xz} = 2.718, 2.459, 2.225$ and 1.234 for $J = 0, 2, 4$ and 6 , respectively, at $E = -40.0$ MeV. For $J = 6$, the value of ρ_{xz} is obtained after smoothing out the fluctuations in $\ln\rho$ by an exponential fitting. An interesting feature we have noticed is that for a fixed value of E and J the level densities ρ_x , ρ_z and $\tilde{\rho}_z$ do not correspond to the same temperature. As we pointed out in section 3.1, for the computation of ρ_z and $\tilde{\rho}_z$ one has to adjust the temperature in such a way that

$$\rho(E(T), J) = \rho(E(T_1), M = J) - \rho(E(T_2), M = J + 1) \quad (20)$$

where, T_1 and T_2 satisfies,

$$E_{M=J}(T_1) = E_{M=J+1}(T_2) \quad (21)$$

We should note that T appearing on the left hand side in eq. 20 is not a quite well defined quantity. Just for the comparison if we define an average temperature $T_{av} = (T_1 + T_2)/2$, we find that the T_{av} in ρ_z and $\tilde{\rho}_z$ for a fixed E and J are quite close. However, looking into the figure 1 we can say that the T_{av} is quite different from the corresponding T in $\rho_x(E, J)$ at low temperatures. Let us now turn towards the discussion on the unique feature for the level densities corresponding to $J = 6$. Figures 3 and 4 show large fluctuations in the values of ρ_x and ρ_z , respectively, for $J=6$. We find that it is quite difficult to adjust the value of ω in the r.h.s. of eq. (13) for $\langle J_x \rangle = \sqrt{6(6+1)}$ as well as $M = 7$ upto a desired precision of 10^{-4} . In principle, one may achieve this precision by varying ω in a very small steps. However, it must be clear from eqs. (9) and (10) that for computing the partition function, each time for a fixed value of ω and T one has to diagonalize the one-body Hamiltonian h'^ω at mesh points in the space of quadrupole variables (see sec. 3.1). So, with a smaller increment in ω it would take enormous computer time to reach a desired value of $\langle J_x \rangle$ and M . We therefore fix the accuracy to be $\sim 10^{-1}$. The fluctuations in the values of the level densities ρ_x and ρ_z are indicative of numerical difficulty in adjusting ω for $J = 6$. For example, the points a and b in the figure 3 correspond to the angular momentum $J_a = 6.15$ and $J_b = 5.85$, respectively.

(ii) ^{64}Zn

The level density

Figure 6 shows the dependence of inverse level density parameter K on the temperature and angular momentum. The curves in this figure essentially depict a similar trend as that for the case of ^{24}Mg in Fig. 2, i.e, a decreases with temperature and angular momentum. However, one may note that the values of K are higher for ^{64}Zn . This feature is consistent with a recent

theoretical[29] investigation on the temperature and mass dependence of the level density parameter in a realistic model. Furthermore, we observe from the figures 2 and 6 that K increases almost linearly with temperature even at low T . On the other hand, in ref. [29] it is shown that for $A = 40$ and 60 it remains more or less a constant up to $T \sim 1.5$ MeV and then increases slowly. It is clear from the ref.[30] that K becomes almost a constant at low temperatures only when the frequency dependent effective mass (m_ω) is introduced to simulate the effects of collectiveness. At high temperatures, the influence of effective mass disappears, i.e m_ω/m approaches unity. This reveals the fact that at low temperatures, the pairing correlations and RPA corrections should be included in the present approach. It must be mentioned here that a similar plot for the parameter K has been shown in ref. [31] for ^{166}Er using a finite temperature mean field approximation. It is found that the value of K is independent of temperature as well as angular momentum for $0.5 < T < 1.4$ MeV. At low temperature, $T < 0.5$ MeV there is a decrease in K at all spins. However, the variation of K for $T > 1.4$ MeV is quite close to as shown presently in figures 2 and 6.

In figures 7 and 8 we have displayed the level densities ρ_x and ρ_z varying with energy at fixed values of angular momenta. As in the case of ^{24}Mg , here too we find that the J_x -cranking yields higher values of the level density, i.e. $\rho_x > \rho_z$. For $J = 0$, difference between the values of ρ_x and ρ_z is very small compared to that for $J \neq 0$. However, the percentage difference, i.e $(\rho_x - \rho_z)/\rho_x$ is smaller for ^{64}Zn .

So far we have only studied the behaviour of the level densities obtained by applying the SPA in different cranking schemes. Apart from the fact that SPA is not applicable at very low temperatures, the present cranking approach shows limitations at low spins also. For this purpose we compare the J_x -cranking results for level density in ^{24}Mg with the one reported

recently[17] for ^{20}Ne using an exact angular momentum projection within SPA. As expected, we find the large difference between the results for $J = 0$. However, for $J \geq 4$, the present results seem to be reasonable. On the other hand one would expect that the level density based on other cranking along z-axis should be quite realistic even at low spins. It is surprising that the value of the level densities for $J = 0$ are the highest even in the schemes (ii) and (iii). This may be the indicative of the necessity of an exact angular momentum or K projection at low spins. Finally we would like to add that the level density versus energy plots discussed above provide only an easy means to compare the results obtained using different schemes. However, for a more realistic comparison one needs to study the variation of level density with the excitation energy. For example, in figure 9 we have plotted ρ_x vs excitation energy E^* (obtained from the use of eqs. (18) and (17)) for fixed values of angular momenta. Unlike in figures 7 and 8, we find that for $E^* = 5 - 10$ MeV (about neutron separation energy i.e. very low T), the level density at $J = 16$ is slightly higher than that for $J = 8$. But at higher energies ($T > 1$ MeV) $J = 16$ curve falls below that for $J = 8$ and there is a tendency of saturation for all the curves.

4. Summary and conclusions

We have investigated the angular momentum dependence of the level density of ^{24}Mg and ^{64}Zn using static path approximation method with a quadrupole interaction Hamiltonian in three different cranking schemes: (i) cranking about x-axis, (ii) cranking about z-axis and (iii) cranking about z-axis including orientation fluctuation corrections (only for ^{24}Mg). In a comparative study we make the following observations.

For ^{24}Mg we find that qualitatively $\rho_z = \tilde{\rho}_z$ for $J = 0$ and 2, with $\tilde{\rho}_z < \rho_z$ for $J = 4$ and 6. This may be an indication of the importance of the

orientation fluctuation at high spins. However, we also notice that at high energy, e.g. $E(T) > -42$ MeV even at $J = 6$ $\tilde{\rho}_z \approx \rho_z$. On the other hand $\rho_x > \rho_z$ for $J < 6$ and $\rho_x \approx \rho_z$ at $J = 6$. As expected, this shows that x-axis cranking may be a good approximation at high spins only.

Now coming to the relatively heavier nucleus ^{64}Zn we find that $\rho_x \approx \rho_z$ at all spins $J = 0 - 28$. Thus for a heavy nucleus and particularly at high spins the x-axis cranking may provide a good prescription for the computation of spin dependent level densities. Due to computation limitations we have not yet included the orientation fluctuation correction for ^{64}Zn . We propose to investigate on this in our next calculation. We prefer to incorporate orientation fluctuation corrections in the x-axis cranking scheme as it also provides us with an opportunity to compute moment of inertia as $\mathfrak{I} = \langle J_x \rangle / \omega$. Recently [19] we have studied the variation of \mathfrak{I} as a function of spin and temperature for ^{64}Zn .

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Table 1: Single particle energies (in MeV) for the spherical orbits present in the model space for ^{24}Mg and ^{64}Zn .

Model space for ^{24}Mg		Model space for ^{64}Zn	
spherical orbit	sp energy	spherical orbit	sp energy
$1s_{1/2}$	-4.03	$1p_{1/2}$	-8.3
$0d_{3/2}$	0.08	$1p_{3/2}$	-10.2
$0d_{5/2}$	-5.00	$0f_{5/2}$	-8.8
-	-	$0f_{7/2}$	-14.4
-	-	$0g_{9/2}$	-4.4

Figure 1: Variation of energy with temperature and the quantum number M . The points A and B on the horizontal dashed line corresponding to $E = -47.5$ MeV indicate the change in temperature with M changing from 2 to 3.

Figure 2: Temperature versus the inverse level density parameter K using eq. (18) for $J = 0, 2, 4$ and 6 . The level density parameter a decreases with increase in temperature as well as angular momentum.

Figure 3: The J_x -cranking results for level density as a function of energy for a few values of angular momenta, $J = 0, 2, 4$ and 6 . Fluctuations in $\ln \rho$ at $J = 6$ occur due to the numerical difficulty for adjusting ω in r.h.s. of eq. (13) up to a reasonable accuracy. For example, the points a and b correspond to the angular momentum $J_a = 6.15$ and $J_b = 5.85$, respectively.

Figure 4: Level density of ^{24}Mg as a function of energy with its angular momentum dependence extracted using eq. (14).

Figure 5: Same as figure 4 with orientation fluctuation correction.

Figure 6: Inverse level density parameter K as a function of temperature at fixed values of angular momenta, $J = 0, 8, 16$ and 28 for ^{64}Zn .

Figure 7: Level density of ^{64}Zn as a function of energy at fixed values of angular momenta obtained in J_x -cranking.

Figure 8: Same as figure 7, with angular momentum dependence extracted using eq. (14).

Figure 9: Level density as a function of excitation energy E^* (obtained from eqs. (17) and (18)) for fixed values of angular momenta.